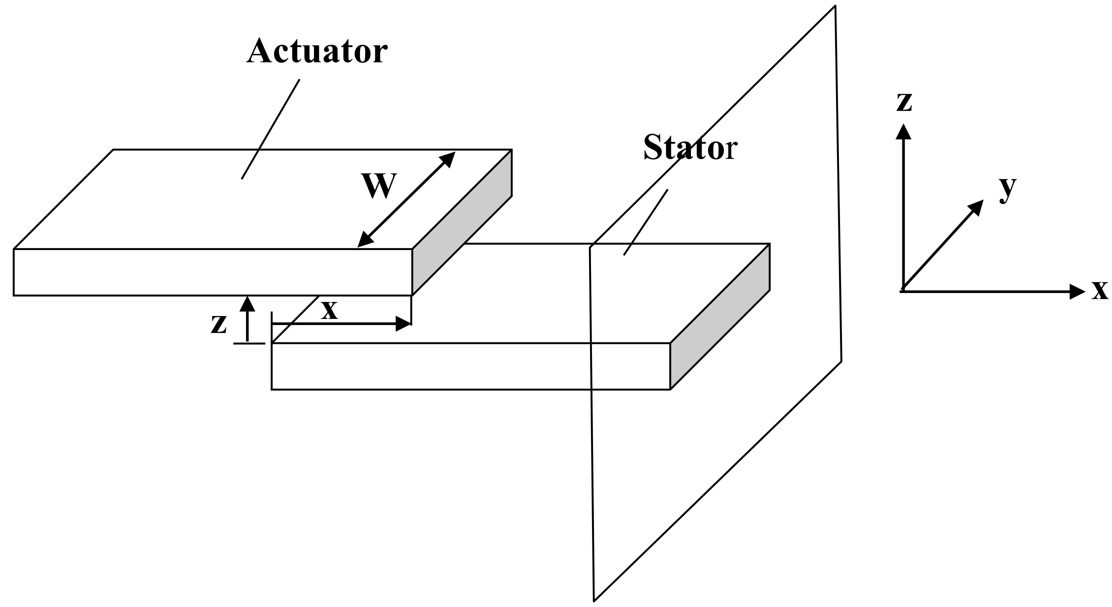
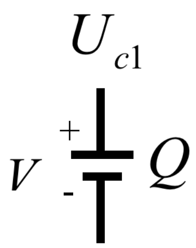
Lecture 8-2 Design Comb-drive Actuator through Cronos/MUMPS Process: I

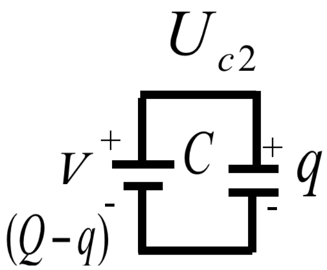
◆ Electrostatic actuator/sensor [1]



1. **Fundamental**

   a. Energy in a battery-capacitor system





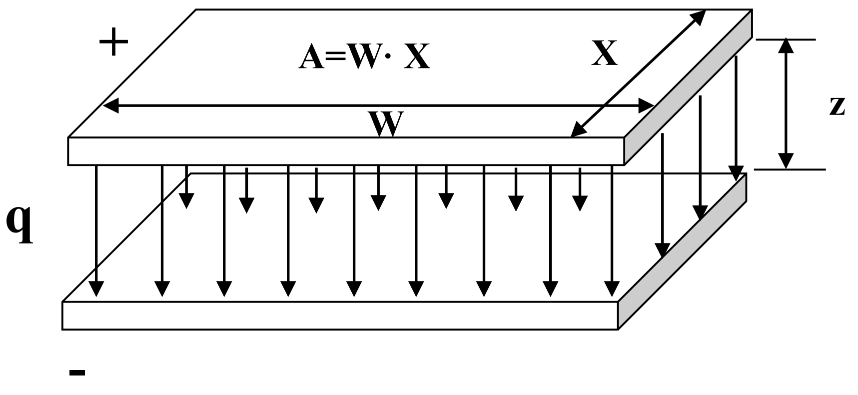
$$U_{c1} = VQ \quad \textbf{(8-1)}$$

$$\begin{aligned} U_{c2} &= U_{c2,b} + U_{c2,c} \\ &= V(Q-q) + \int_0^q V dq \\ &= V(Q-CV) + \boxed{\frac{1}{2}CV^2} \quad (q = CV) \quad (8-2) \\ &= \boxed{VQ - \frac{1}{2}CV^2} \quad (8-3) \end{aligned}$$

Energy stored in capacitance

System energy reduced by $\frac{1}{2}CV^2$

b. Capacitance of a flat plate



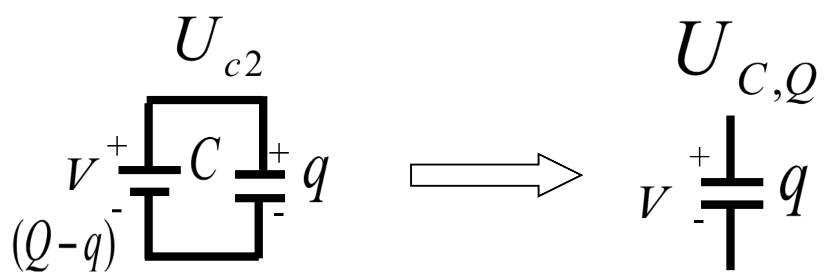
Neglect end effect

$$C = k\frac{\varepsilon_0 A}{z} = k\frac{\varepsilon_0 WX}{z} \qquad \textbf{(8-4)}$$

Here *k*: Dielectric constant, for vacuum: 1.0, air: 1.00059, Pyrex: 5.6, Teflon: 2.1, water 80.

ε_0: Permittivity of free space: $8.85e^{-12}$ C^2/Nm^2

2. Constant Charge mode



Energy stored:

$$U_c = \frac{1}{2}CV_0^{\,2} = \frac{q_0^2}{2C} = \frac{1}{2k\varepsilon_0}\left(\frac{z}{WX}\right)q_0^2 \qquad \textbf{(8-5)}$$

a. Fix gap z, move in X direction

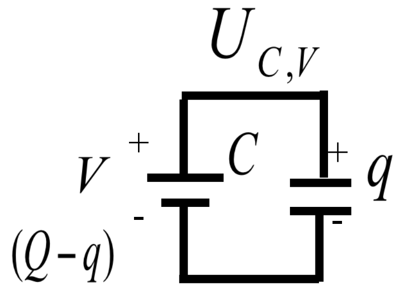
$$ {}_{Q}F_{x} = \frac{-\partial U_C}{\partial X} = \frac{1}{2k\varepsilon_0}\left(\frac{z}{WX^2}\right)q_0^2 \qquad \textbf{(8-6)} $$

b. Fix X position, move in z direction

$$ {}_{Q}F_{z} = \frac{-\partial U_C}{\partial z} = \frac{1}{2k\varepsilon_0}\left(\frac{1}{WX}\right)q_0^2 \qquad \textbf{(8-7)} $$

Constant force!!

3. Constant Voltage mode



Energy stored:

$$ U_{c,V} = VQ_0 - \frac{1}{2}CV_0^2 = VQ_0 - \frac{1}{2}\left(\frac{k\varepsilon_0 WX}{z}\right)V_0^2 \qquad \textbf{(8-8)} $$

a. Fix gap z, move in X direction

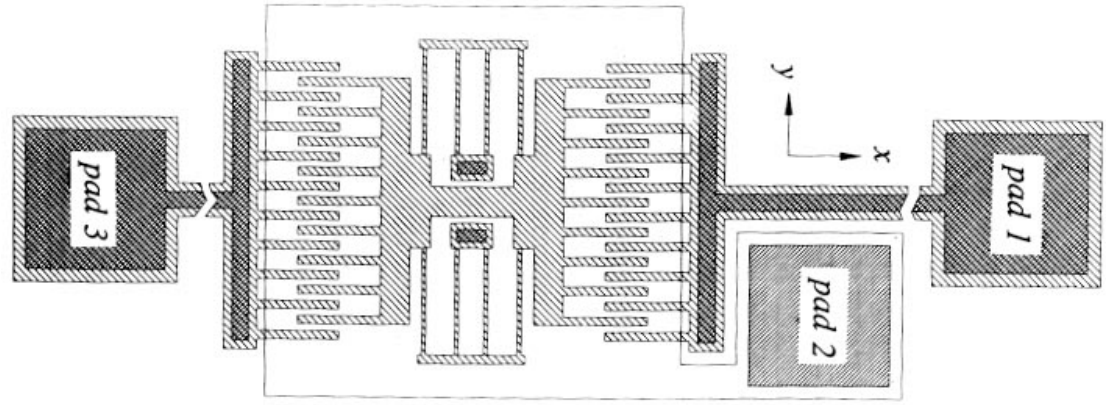
$$ {}_{V}F_{x} = \frac{-\partial U_C}{\partial X} = \frac{1}{2}\left(\frac{k\varepsilon_0 W}{z}\right)V_0^2 \qquad \textbf{(8-9)} $$

Constant force!!

c. Fix X position, move in z direction

$${}_V F_z = \frac{-\partial U_C}{\partial z} = \frac{-1}{2}\left(\frac{k\varepsilon_0 WX}{z^2}\right)V_0^2 \qquad \textbf{(8-10)}$$

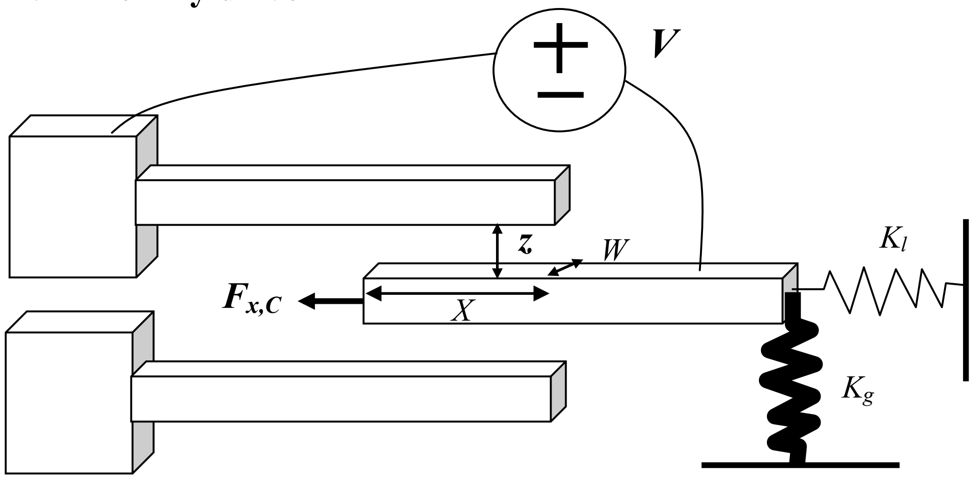
◆ **Comb drive actuator (Tang et. Al., MEMS'89)[1]**



Layout of a linear resonant plate with comb structures on both ends and a 50 μm long folded-beam on each side.

1. **Driving force and displacement**

   a. **Laterally driven**

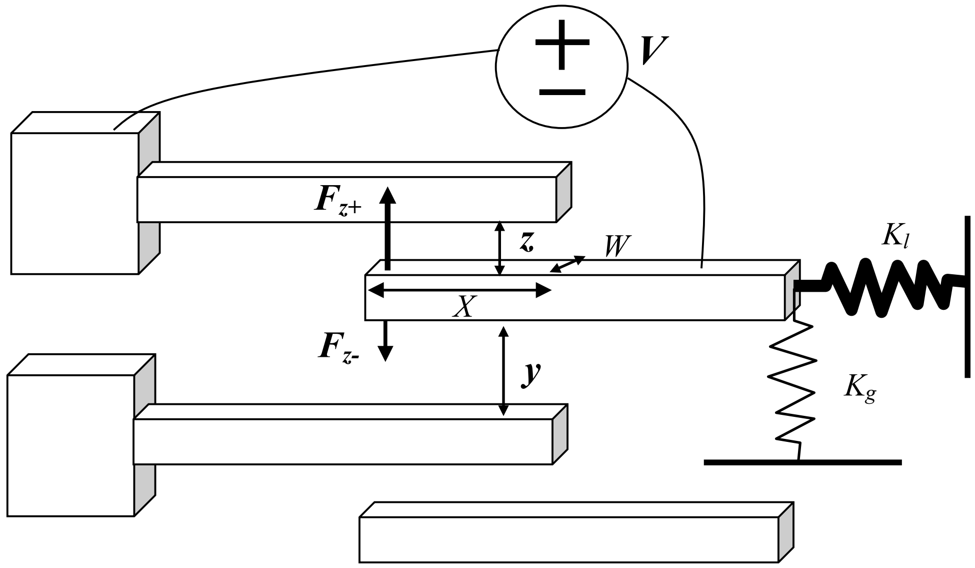


$$C_{total} = N\varepsilon_0 \frac{WX}{z} \quad \textbf{(For W>>Z, N: \# of gaps)} \qquad \textbf{(8-11)}$$

$$F_{x,C} = \frac{1}{2} N\varepsilon_0 V^2 \frac{W}{z} \qquad \textbf{(8-12)}$$

i. Large displacement
ii. Small force
iii. Constant force

b. Gap-closing driven



$$C_{total,z} = N_z \varepsilon_0 \frac{WX}{z}$$ **(For W>>z, N_z: # of closer gaps) (8-13)**

$$C_{total,y} = N_y \varepsilon_0 \frac{WX}{y}$$ **(For W>>y, N_y: # of farer gaps) (8-14)**

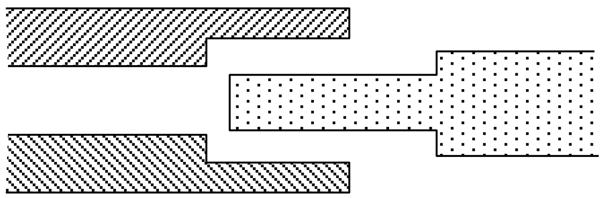
$$F_z = F_{z+} - F_{z-} = \frac{-1}{2} N_z \left(\frac{k\varepsilon_0 WX}{z^2}\right) V^2 - \frac{-1}{2} N_y \left(\frac{k\varepsilon_0 WX}{y^2}\right) V^2$$
$$\approx \frac{-1}{2} N_z \left(\frac{k\varepsilon_0 WX}{z^2}\right) V^2 \qquad (for\ z > y)$$

(8-15)

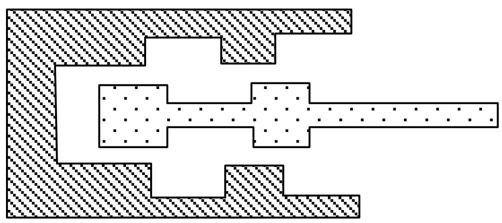
i. Nonlinear force
ii. Small displacement

2. To get larger force

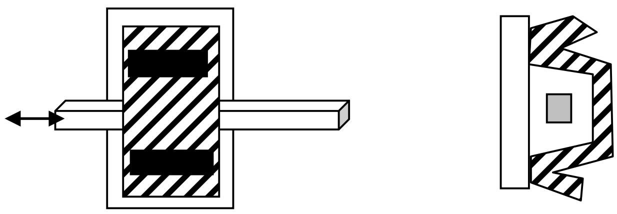
a. **Gap closer-better lithography resolution**
b. **Higher aspect ratio- using LIGA, SCREAM, Deep Silicon RIE, Thick PR...increase W**
c. **Special initial fixing technique [2]…decrease Z**



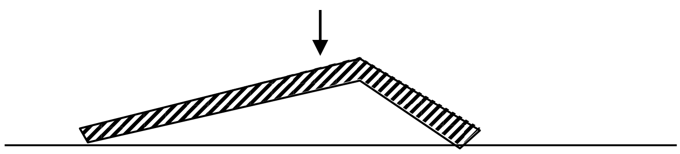
d. **Increase capacitive area: tooth like surface**



e. **Increase capacitive area: Cylinder type surface**

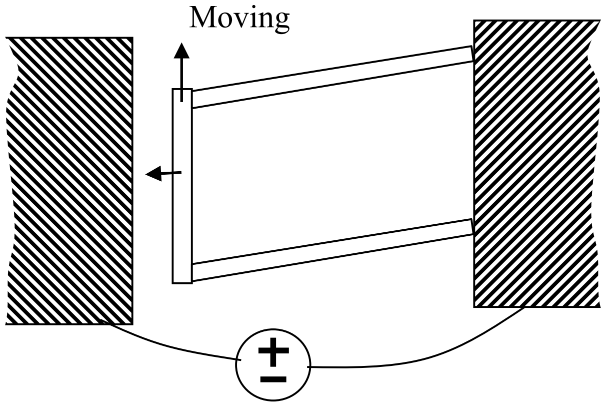


f. **Using buckling effect-scratch actuator**

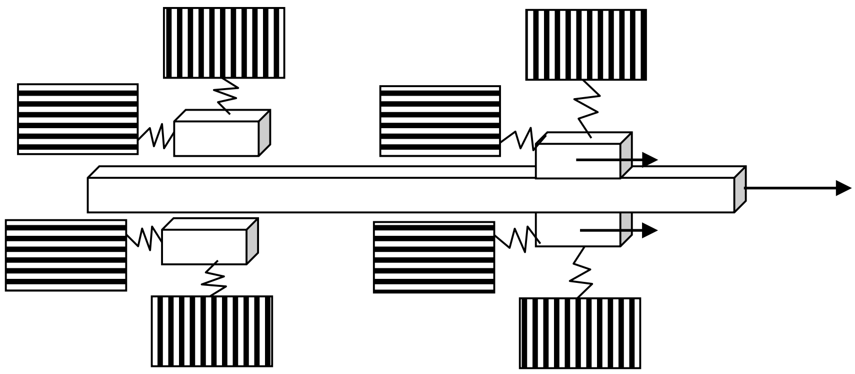


3. To get larger Displacement

a. Laterally driven+gap closing [3]

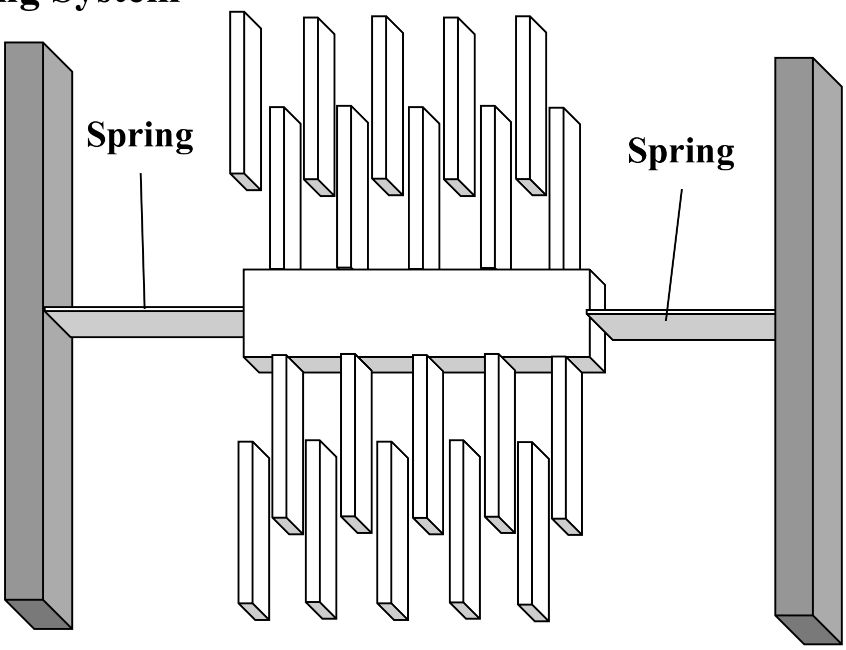


b. Inch worm

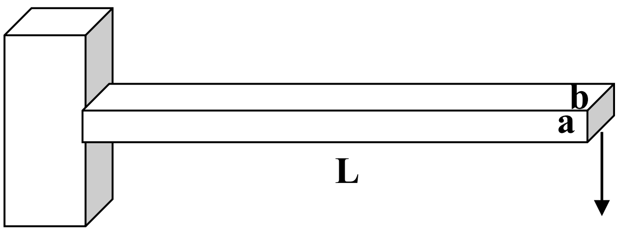


c. Transverse motion=> rotational motion (Sandia national lab, comb drive-> gear set)

◆ Spring System

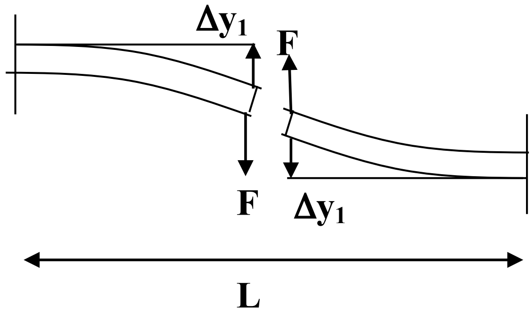


1. Cantilever beam



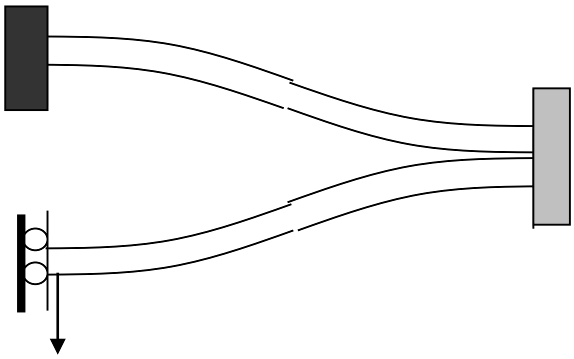
$$K(a,b,L) = \frac{F}{\Delta y} = \frac{Ea^3 b}{4L^3} = K_0 \qquad \textbf{(8-16)}$$

2. Two ends constrained beam:



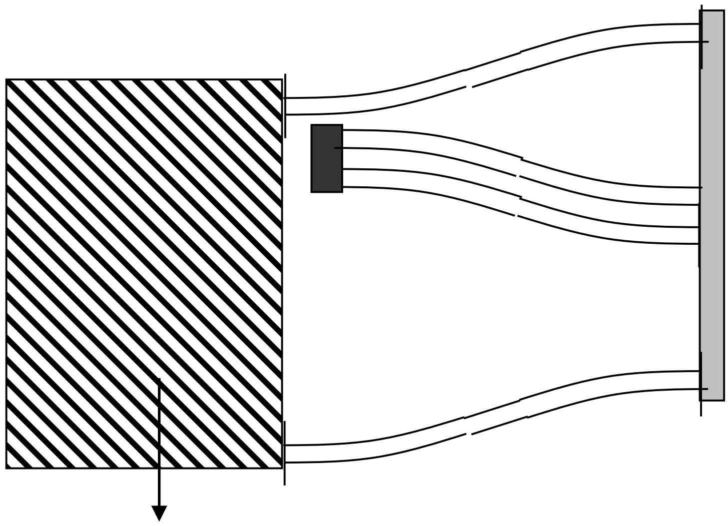
$$k_1 = \frac{F}{2\Delta y_1} = \frac{1}{2}k(a,b,\frac{L}{2}) = 4K_0 \quad \textbf{(8-17)}$$

3. Crab lag



$$k_2 = \frac{1}{2}K_1 = 2K_0 \quad \textbf{(8-18)}$$

4. Folded beam



$$k_{fold} = 2K_2 = 4K_0 \quad \textbf{(8-19)}$$

◆ Viscous Damping [4]

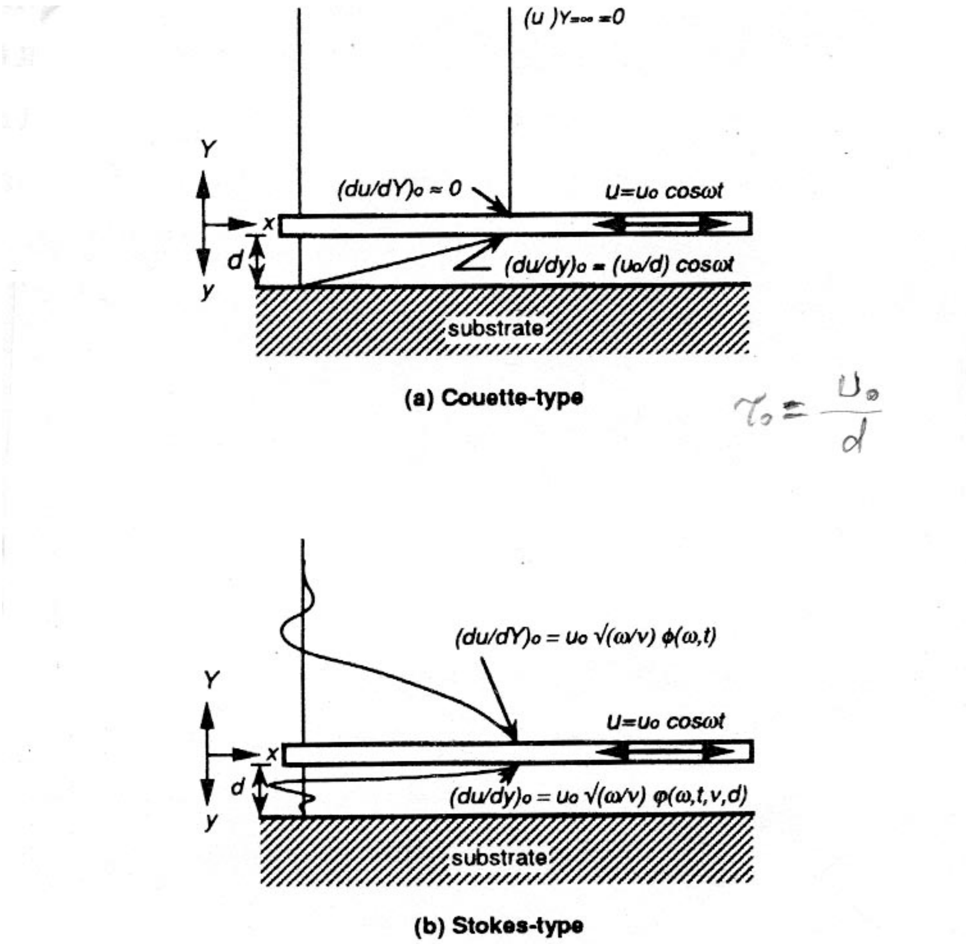


Fig. 2. Velocity profiles in the fluid layers induced by a laterally oscillating infinite plate.

Couette-type damper:

$$Q_{cd} = \frac{d}{\mu A}\sqrt{MK} \qquad \textbf{(8-20)}$$

Q: quality factor, $Q=\omega_n M/b$

Stokes-type damper:

$$\frac{1}{Q_s} = \frac{1}{Q_{sd}} + \frac{1}{Q_{s\infty}} + \frac{1}{Q_{sc}} \qquad \textbf{(8-21)}$$

total=(finger to ground) + (finger to ∞) + (finger to finger)

where $$Q_{sd} = Q_{cd} \frac{(\cosh 2\beta d - \cos 2\beta d)}{\beta d(\sinh 2\beta d + \sin 2\beta d)} \qquad \textbf{(8-22)}$$

$$Q_{s\infty} = Q_{cd} \frac{1}{\beta d} \qquad \textbf{(8-23)}$$

$$Q_{sc} = Q_{cd} \frac{A_{finger}}{A} \frac{(\cosh 2\beta z - \cos 2\beta z)}{\beta d(\sinh 2\beta z + \sin 2\beta z)} \qquad \textbf{(8-24)}$$

where

$$\beta = \sqrt{\frac{\omega}{2\nu}}, \quad \omega : osillation\ frequency, \nu : dynamic\ viscousity$$

$$z : finger\ gap, Q : quality\ factor \equiv \frac{\omega M}{b}$$

◆ **Dynamic Response**

Laterally driven:

$$M\ddot{X} + b\dot{X} + K(X - X_0) = F_{x,C} = \frac{1}{2} N\varepsilon_0 V^2 \frac{W}{z} = cons\tan t$$

(8-25)

Gap closing:

$$M\ddot{z} + b\dot{z} + K(z - z_0) = F_{z,C} = \frac{-1}{2} N_z \left(\frac{k\varepsilon_0 WX}{z^2}\right) V^2$$

(8-26)

Frequency response

$$\omega = \sqrt{\frac{K}{M}} \qquad \textbf{(8-27)}$$

Reference

[1] William C. Tang, Tu-Cuong H. Nguyen, and Roger T. Howe, "Laterally Driven Polysilicon Resonant Microstructres", Proceedings of MEMS '89, pp.187-193, Feb., 1989.

[2] T. Hirano, T. Furuhata, K. J. Gabriel, and H. Fujita, "Operation of Sub-micron Gap Electrostatic Comb-drive Actuators", Proceedings of MEMS'91, pp. 873-876, Feb. 1991.

[3] Reid A Brennen, Martin G. Lim, Albert P. Pisano, and Alan T. Chou, "Large Displacement Linear Actuator", Technical Digest IEEE Solid-State Sensors and Actuators, pp. 135-139, June, 1990.

[4] Young-Ho Cho, Albert P. Pisano, and Roger T. Howe, "Viscous Damping Model for Laterally Oscillation Microstructures", Journal of MEMS, Vol. 3, No. 2, pp. 81-86, June, 1994.