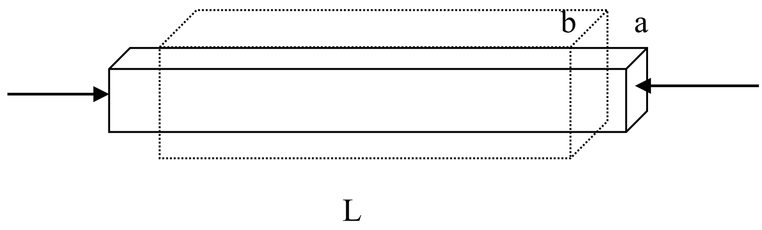
Lecture 7-2 MOSIS/SCNA Design Example-

Piezoresistive type Accelerometer II

◆ Piezoresistivity



Consider a conductive block of dimension a×b×L as shown in the figure. If a current is passed through the block parallel to the direction L, the resistance of the block is given by

$$R_0 = \frac{\rho L}{ab} \quad \textbf{(7-1-23)}$$

where ρ is the resistivity of the block material. When an inward force F is applied against the two ends of the block, this generates a stress $\sigma = -F/L^2$**, and the block deforms. The amount of deformation is a function of the Young's modulus E and the Poisson's ratio ν of the material. The change in dimensions of the block are given by**

$$\Delta L = \varepsilon L = \frac{\sigma L}{E} \qquad \Delta a = \varepsilon \nu a = \frac{\sigma \nu a}{E} \qquad \Delta b = \varepsilon \nu b = \frac{\sigma \nu b}{E} \quad \textbf{(7-1-24)}$$

This dimension change results in a new value for the resistance:

$$R_\varepsilon = \frac{\rho(1-\varepsilon)L}{(1+\varepsilon\nu)a(1+\varepsilon\nu)b} \approx [1-(1+2\nu)\varepsilon]R_0 \quad \textbf{(7-1-25)}$$

The gage factor G_{gauge} is defined to be the change in resistance per change in strain:

$$G_{guage} \equiv -\frac{\Delta R / R_0}{\varepsilon} \qquad \textbf{(7-1-26)}$$

The gauge factor from Eqn. (7-1-25) is G=1+2ν. The Poisson's ratio for most materials is roughly 0.3, giving a typical gauge factor for most conductors of $G_{gauge} \cong 1.6$. e.g. a 1% strain will result in a 1.6% change in resistance.

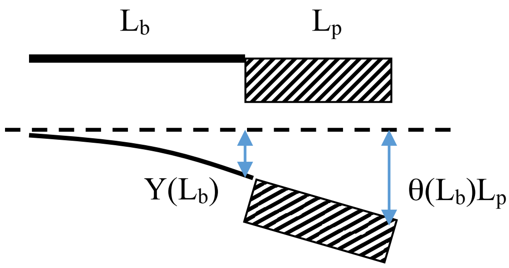
In addition to this shape-based change in resistance, some materials exhibit a substantial change in resistance due to a strain-induced change in the resistivity of the material. This is known as the piezoresistive effect, and is particularly apparent in silicon crystals, where the gauge factor due to piezoresistivity can be as large as 200 in magnitude. The gauge factors of n-type and p-type silicon have opposite sign, with n-type being negative. Due to a variety of effects, the gauge factor of polysilicon films is typically an order of magnitude lower than single crystal films, with typical values of 20 to 30 for p-type silicon, and –15 to –25 for n-type.

The average strain for finite length of piezoresistive material sitting on top of the beam, extending a length L_g from the base of the beam. The strain varies linearly along the beam, and is zero at the tip and maximum at the base, so the average stain in the piezoresistor is given by:

$$\varepsilon_{ave} = (1 - \frac{L_g}{2L_b})\varepsilon_{\max} \qquad \textbf{(7-1-27)}$$

Combining Eqn. 7-1-22, 26, 27, we can have

$$\frac{\Delta R}{R} = G_{gauge}(1 - \frac{L_g}{2L_b})\frac{3}{2}\frac{ya}{L^2} \qquad \textbf{(7-1-28)}$$



◆ **Maximum deflection**

From Eqn. 7-1-14, the deflection of a cantilever beam:

$$y(x) = \frac{1}{EI}[\frac{Fx^2}{6}(3L - x) + \frac{M_0 x^2}{2}]$$

where F and M are the force and moment applied to the beam. In the above case, F=ma, and M=FL_p/2

$$y(L_b) = \frac{1}{EI}[\frac{FL_b^{\ 3}}{3} + \frac{ML_b^{\ 2}}{2}] = \frac{F}{EI}[\frac{L_b^{\ 3}}{3} + \frac{L_b^{\ 2}L_p}{4}] \qquad \textbf{(7-1-29)}$$

The tip of the proof mass deflects an additional amount determined by the angle of the tip of the support beam (assume the proof mass is a rigid body without any deflection under the acceleration force)

$$\theta(L_b) = \frac{F}{EI}[\frac{L_b^2}{2} + \frac{L_p L_b}{2}] \qquad \textbf{(7-1-30)}$$

The deflection of the tip of the proof mass is

$$y(L_b + L_p) = y(L_b) + \theta(L_b)L_p \qquad \textbf{(7-1-31)}$$

◆ Resonant frequency

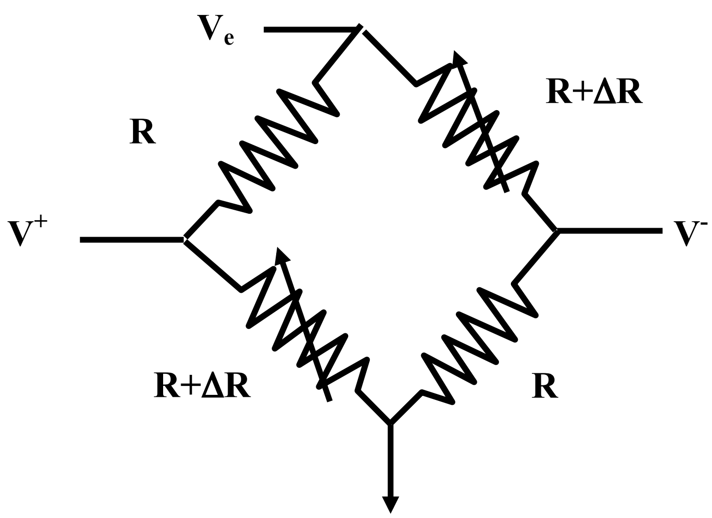
From the analysis of the dynamic of the beam, the natural frequency is

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{\frac{Ea^3b}{4L_b^3}}{WL_p t\rho}} \qquad \textbf{(7-1-32)}$$

◆ Responsivity

$$\text{Re}\,sponsivity \equiv \frac{output \quad signal}{input \quad physical \quad quantity}$$

$$\Rightarrow R_{accelerometer} = \frac{output \quad Voltage}{input \quad acceleration}$$



From the above Wheatstone bridge

$$\Delta V = V^{+} - V^{-} = \frac{R+\Delta R}{2R+\Delta R}V_e - \frac{R}{2R+\Delta R}V_e = \frac{\Delta R}{2R+\Delta R}V_e \approx \frac{\Delta R}{2R}V_e$$

(7-1-33)

From Eqn. (7-1-26)

$$\frac{\Delta R}{R} = G_{gauge}\varepsilon = G_{gauge}(1-\frac{L_g}{2L})\varepsilon_{\max}$$

$$= G_{gauge}(1-\frac{L_g}{2L})\frac{zM_{\max}}{EI}$$

$$= G_{gauge}(1-\frac{L_g}{2L})\frac{z[(ma_{ace})(\frac{L_p}{2}+L_b)]}{E(\frac{a^3 b}{12})}$$ **(7-1-34)**

Combine Eqns. 7-1-33 and 34, we can have

$$R_{acce} = \frac{\Delta V}{a_{ace}} = 6V_e G_{gauge}(1-\frac{L_g}{2L_b})\frac{z(tWL_p\rho)(\frac{L_p}{2}+L_b)}{Ea^3 b}$$

$$\Rightarrow R_{acce} \approx 3V_e G_{gauge}\frac{z}{Ea^3 b}(tWL_p^2\rho)$$

(7-1-34)

◆ Noise

There are several sources of noise in this device.

1. Johnson Noise:

The resistors in the Wheatstone bridge each contribute noise power of 4kT per Hertz of bandwidth. Johnson noise comes from thermal agitation of electrons within a resistance, and it sets a lower limit on the noise present in a circuit. The corresponding effective voltage noise is related to power by the usual relationship:

$$\text{Power} = v_n^2 / R, \text{ or}$$

$$\bar{v}_n = \sqrt{4kTR\Delta f} \qquad \textbf{(7-1-35)}$$

Where Δf is the bandwidth of interest.

Johnson noise is also referred to as thermal noise, resistance noise or white noise. It is independent of the composition of the resistance, and the frequency distribution of thermal noise power is uniform. The instantaneous amplitude for thermal noise has a Gaussian, or normal, distribution. The average is zero and the RMS value is given above.

The thermal noise generated by any arbitrary connection of passive elements is equal to the thermal noise that would be generated by a resistance equal to the real part of the equivalent network impedance.

2. TNEA (Thermal noise-equivalent acceleration):

The spring/mass system of the cantilever contributes its own noise to the system, with a noise power of kT/2 in each mode of vibration. This leads to an effective RMS deflection of

$$\frac{1}{2}K\bar{x}_n^2 = \frac{1}{2}kT \qquad \textbf{(7-1-36)}$$

which yields an equivalent RMS acceleration of

$$\bar{a}_n = \sqrt{\frac{4kT\omega_0}{MQ}} \qquad \textbf{(7-1-37)}$$

where ω_0 **is the nature frequency,** ***M*** **is the mass,** ***K*** **is the spring constant, and** $Q = \frac{\omega_0 M}{c}$ **is the quality factor of the spring system.** ***k*** **is Boltzmann's constant (**$1.38e^{-23}$ **joules/°K).**

◆ Sensitivity

In order to compare the relative importance of these noise sources, it is useful to move them from their source to some common location in the signal block diagram. Typically this is the input to the electronic amplifier, or the real input (such as acceleration). In this way we can really compare the effects from different noise sources. For example, we can move all above noises to the input, by which we can have them as "noise equivalent acceleration". Here the "noise equivalent acceleration" is the "sensitivity" of the system, i.e., the minimum input signal we can detect.

◆ Dynamic range

1. Upper limit: Strain limits

For most of the micro-mechanical materials, the fracture strain is roughly 1%. We need to transfer the strain limit into equivalent acceleration.

From Eqn. 7-1-20

$$\varepsilon_{frac}(0,z) = \frac{zM}{EI}$$

$$= \frac{z[(ma_{upper})(\frac{L_p}{2} + L_b)]}{E(\frac{a^3 b}{12})}$$

$$\Rightarrow a_{upper} = \varepsilon(0,\frac{a}{2})\frac{E(\frac{a^3 b}{12})}{\frac{a}{2}[(m)(\frac{L_p}{2} + L_b)]} \qquad \textbf{(7-1-38)}$$

2. Lower limit: Sensitivity

Here is the ”noise equivalent acceleration”.

◆ **Temperature dependency**

TCR (Temperature Coefficient of Resistance)

1. Metal:

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

Material	Resistivity: ρ 10^{-8} Ωm	TCR: α (10^{-3} °K)
Aluminum	2.65	3.9
Copper	1.678	3.9
Nickel	6.8	6.9
Platinum	10.42	3.7
Gold	2.24	3.4

Table adopted from J. Fraden[1].

2. Silicon: very sensitivity to purity

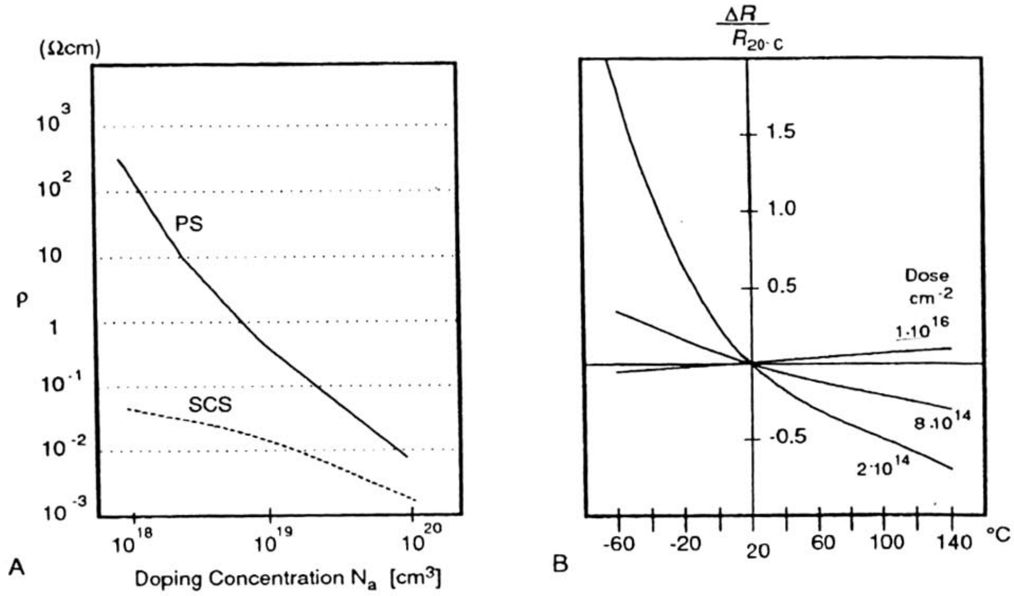


Fig. 3-17.2 Specific resistivity of boron doped silicon (A); temperature coefficient of resistivity of silicon for different doping concentrations (B)

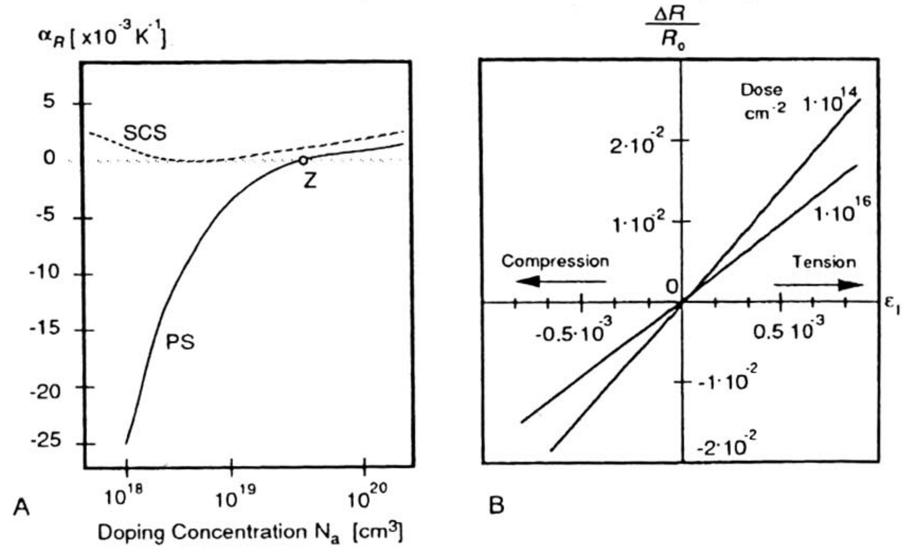


Fig. 3-17.3 Temperature coefficient as a function of doping (A) and piezoresistive sensitivity of silicon (B)

J. Fraden[1]

2. **How to compensate temp. effect?**
   a. **Cancel out through reference or symmetry design.**
   b. **Feedback control through adjustable gain from temp sensor**
   c. **Software adjustment**

◆ Long term stability

1. **Operation Lifetime?**

2. **On shelf life?**

3. **Long-term drift?**

Reference:

1. Jacob Fraden, "AIP Handbook of Modern sensors, Physics, Designs and Applications", American Institute of Physics, 1993.
2. Henry W. Ott, "Noise Reduction Techniques in Electronic Systems", second edition, John Wiley & Sons, Inc., 1988.
3. James M. Gere and Stephen P. Timoshenko, "Mechanics of Materials", fourth edition, PWS publishing company, 1997.
4. Semiconductor Sensors, S.M. Sze, Wiley Inter. Science, 1994.