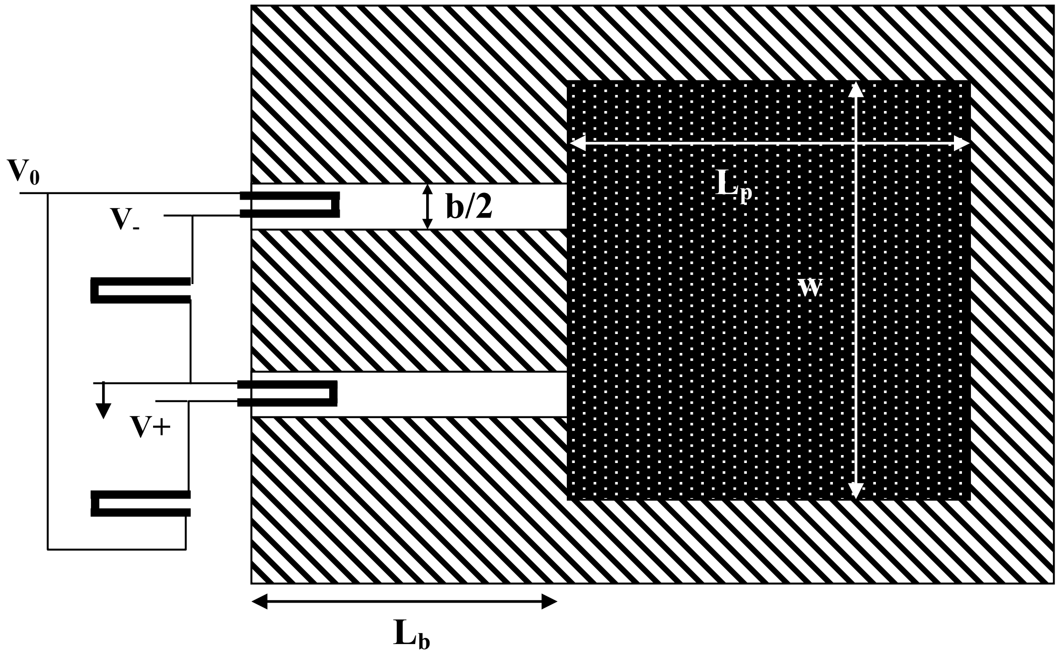
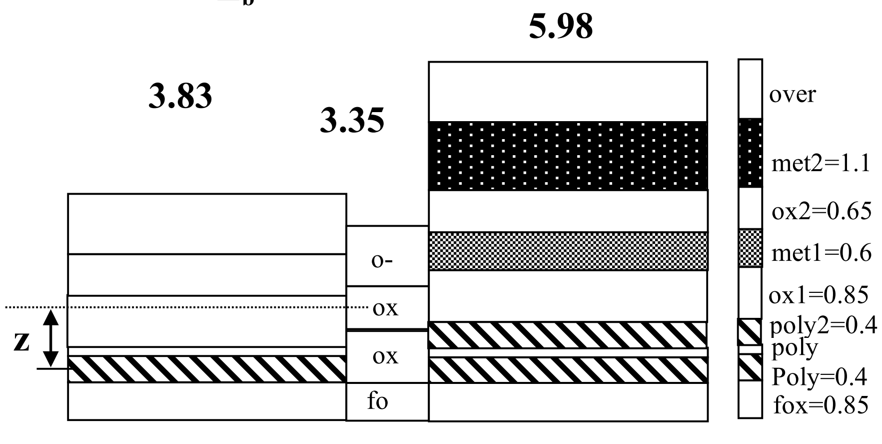
Lecture 7-1 MOSIS/SCNA Design Example-

Piezoresistive type Accelerometer I

◆ Schematic Figure:

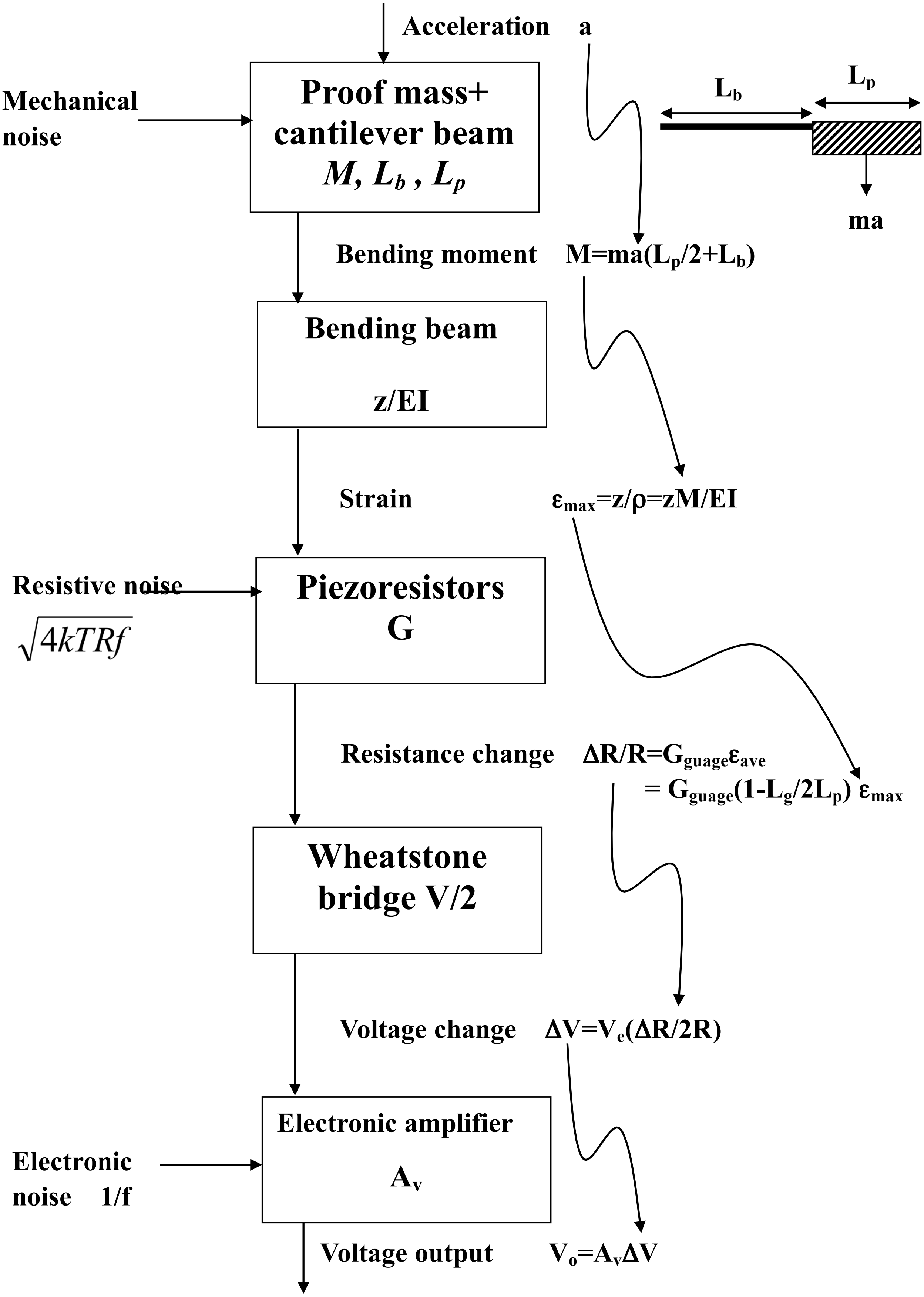




◆ MEMS Accelerometer:

1. **Consists a proof mass and a force detection system**
2. **Bulk micromachining to make proof mass ~mg**
3. **Detection methods: piezoresistive, capacitive, tunneling, acoustic wave, and optical methods.**

- **Signal flow block diagram**



Dynamics

The element for accelerometers and seismometers:

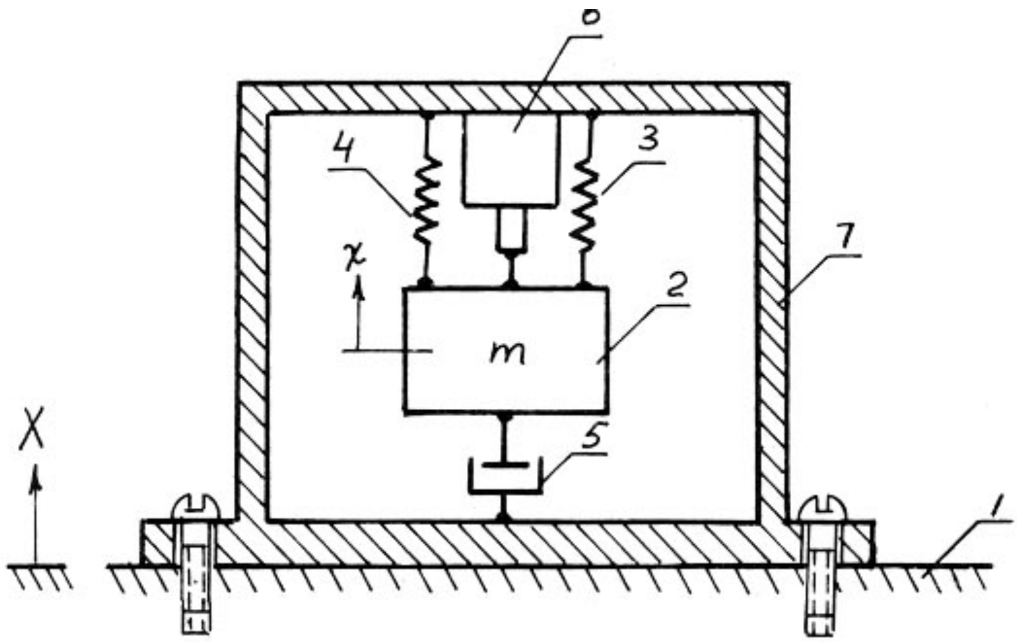


Figure 5.19 Schematic diagram of vibration measuring instrument. *X* = motion to be measured, *x* = motion of seismic mass, 1 = moving support, 2 = seismic mass, 3 and 4 = springs, 5 = dashpot (damper), 6 = motion sensor, 7 = supporting case.

The behavior of the instrument is described by the equation of motion:

$$m\frac{d^2x}{dt^2} + b\left(\frac{dx}{dt} - \frac{dX}{dt}\right) + k(x - X) = 0 \qquad \textbf{(7-1-1)}$$

For m (kg) is the mass of instrument, k (N/m) is the spring constant, b (N s/m) is the friction/damping coefficient, x is the displacement of the mass, X is the displacement of the object.
Z=(X-x) is the mass relative-to-the-case motion.
So we can rewrite Eqn. (1) as

$$m\frac{d^2z}{dt^2}+b\frac{dz}{dt}+kz=m\frac{d^2X}{dt^2} \qquad \textbf{(7-1-2)}$$

(a) If the displacement X is associated with a constant acceleration (like car accident, can be measured by an accelerometer)

$$m\frac{d^2X}{dt^2}=C \qquad \textbf{(7-1-3a)}$$

Then (7-1-2) becomes

$$m\frac{d^2z}{dt^2}+b\frac{dz}{dt}+kz=C \qquad \textbf{(7-1-4a)}$$

If we now replace:

$\omega_n=\sqrt{k/m}$ **=natural frequency of under-damped oscillation, rad/s**

$\varsigma=b/b_c$ **= damping factor, dimensionless**

$b_c=2m\omega_n$**=**$2\sqrt{km}$**=critical damping, kg/s**

$(from: b_c^2-4km=0)$

and solve the above equation by assuming a complex number:

$$z=e^{\gamma t}$$

then

$$\gamma_{\pm}=-\zeta\omega_n\pm\omega_n\sqrt{\zeta^2-1}$$

The solution to the differential equation is thus

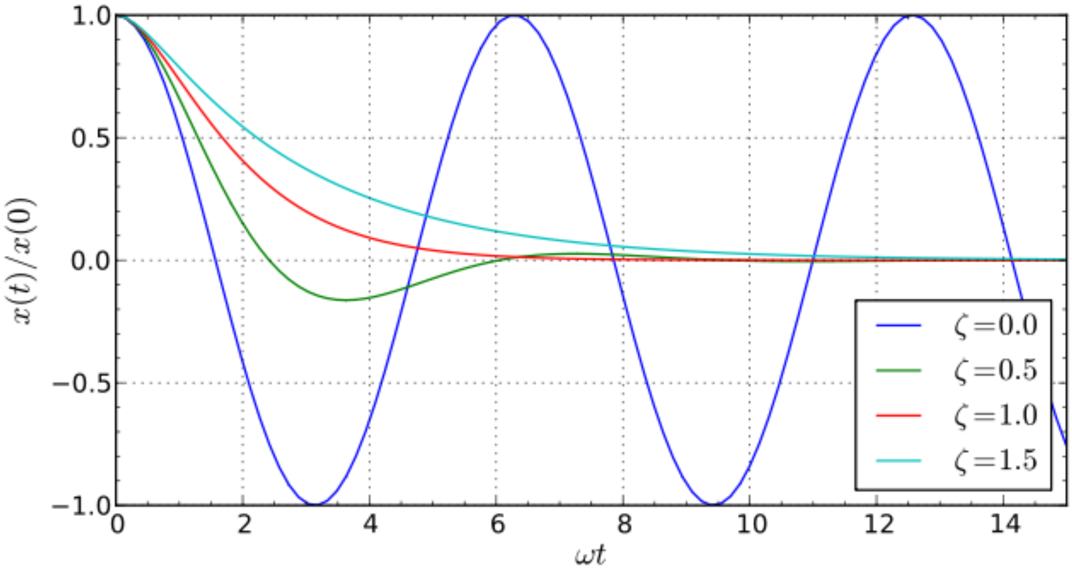
$$z(t)=Ae^{\gamma_+t}+Be^{\gamma_-t}+\frac{C}{k} \qquad \textbf{(7-1-5a)}$$

We can find out the system dynamic behavior:

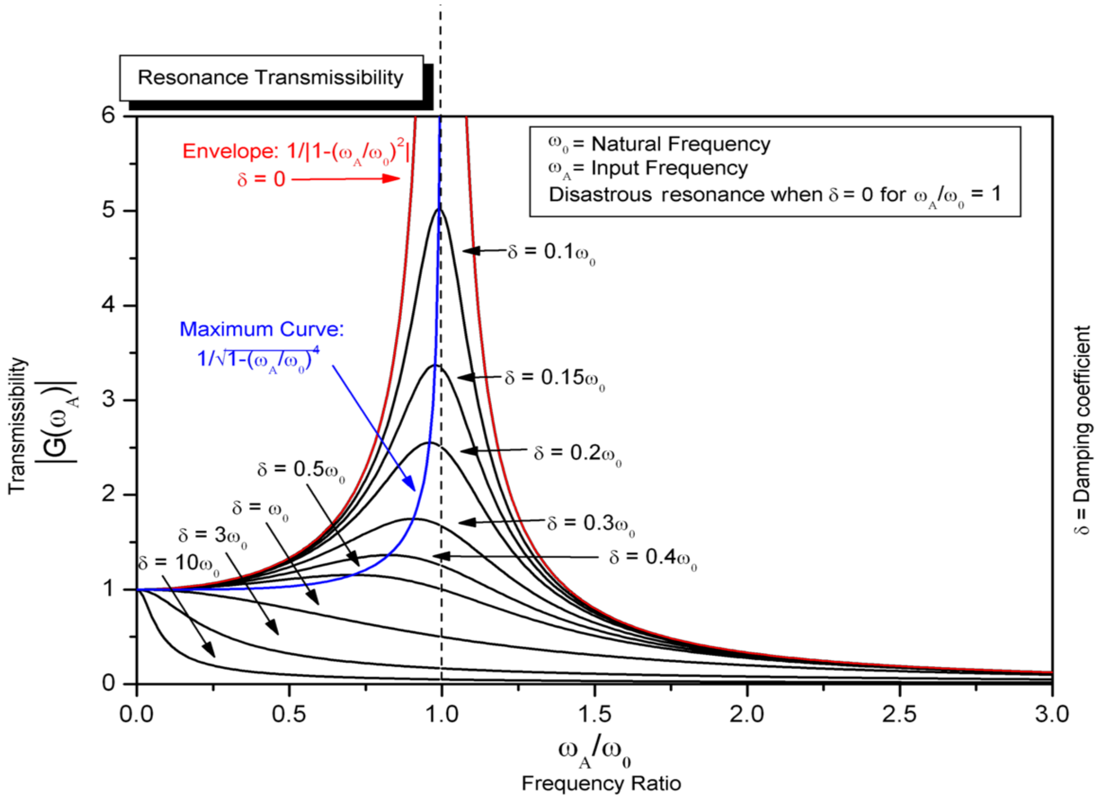
(a) over-damped system: $b_c^2 - 4km > 0$ **or** $\zeta > 1$

(b) critical-damped system: $b_c^2 - 4km = 0$ **or** $\zeta = 1$

(c) under-damped system: $b_c^2 - 4km < 0$ **or** $\zeta < 1$



Time dependence of the system behavior on the value of the damping ratio ζ, for undamped *(blue)*, under-damped *(green)*, critically damped *(red)*, and over-damped *(cyan)* cases, for zero-velocity initial condition.



Steady state variation of amplitude with frequency and damping of a driven simple harmonic oscillator

(b) If the displacement X is a sinusoidal vibration with peak value X_0 (like earthquake, detected by a seismometer):

$$X = X_0 \sin \omega t \tag{7-1-3b}$$

Then Eqn. (7-1-2) will be

$$m\frac{d^2 z}{dt^2} + b\frac{dz}{dt} + kz = mX_0\omega^2 \sin \omega t \tag{7-1-4b}$$

The steady state solution:

$$z = z_0 \sin(\omega t - \phi) \tag{7-1-5b}$$

where

$$z_0 = \frac{mX_0\omega^2}{\sqrt{(k - m\omega^2)^2 + (b\omega)^2}} = \frac{X_0(\frac{\omega}{\omega_n})^2}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2 + (2\varsigma\frac{\omega}{\omega_n})^2}} \tag{7-1-6b}$$

$$\phi = \tan^{-1}\frac{\omega b}{k - m\omega^2} = \frac{2\varsigma\frac{\omega}{\omega_n}}{1-(\frac{\omega}{\omega_n})^2} \tag{7-1-7b}$$

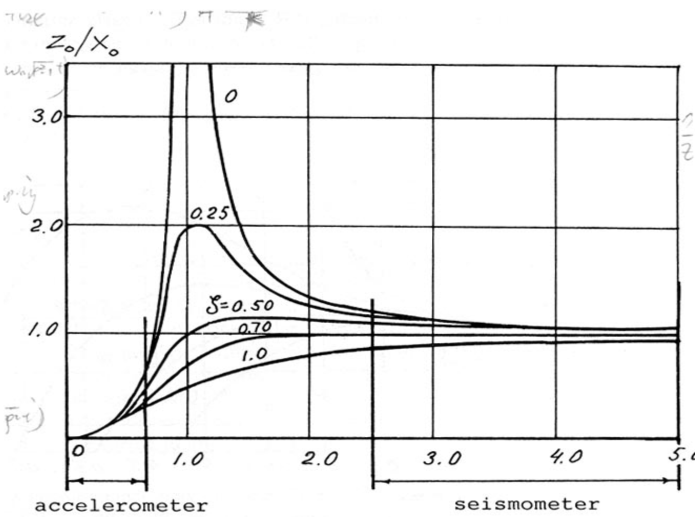
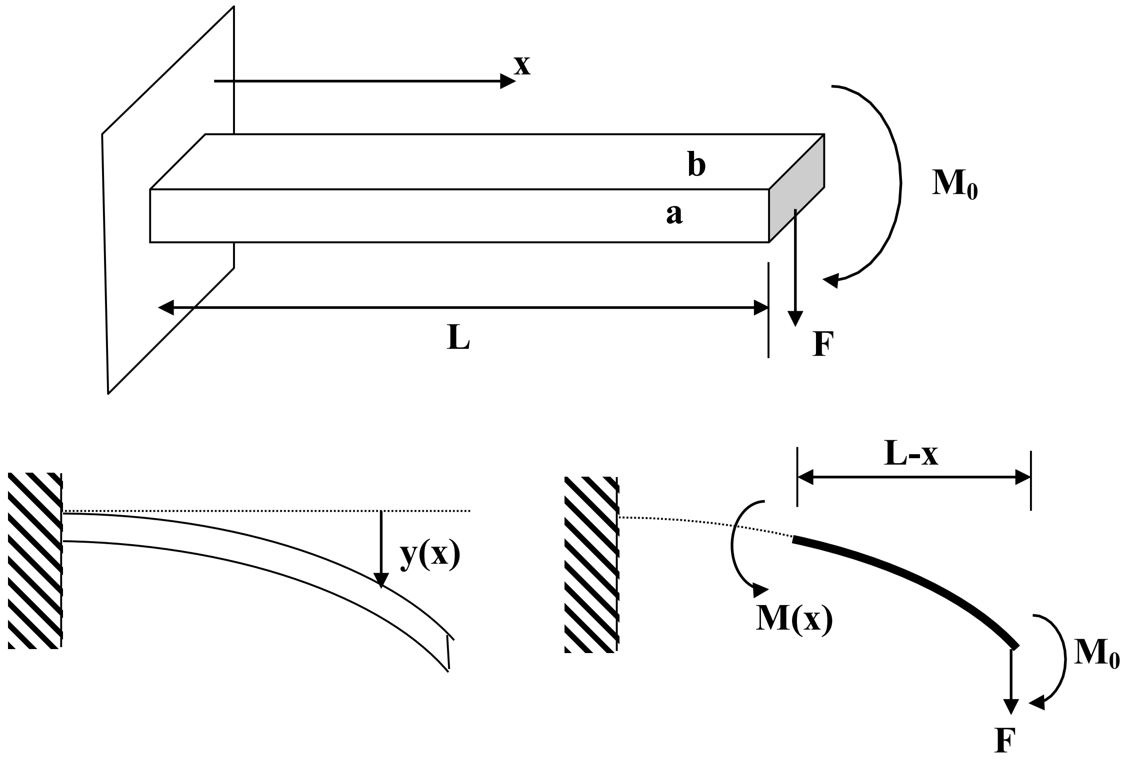


Figure 5.20 Amplitude response of vibration-measuring instruments.

Material Mechanics-Beam Theory



The Moment-Curvature relation for a simple beam is

$$\frac{1}{\rho(s)} = \frac{M(s)}{EI} \qquad \textbf{(7-1-8)}$$

where M is the bending moment, s is the arc length along the beam, ρ is the radius of curvature, E is the Young's modulus of the material, and I is the moment of inertia of the beam cross-section. EI is called the flexural stiffness of the beam. Both E and I may be functions of the arc length, but are constants for an isotropic constant-cross-section beam. For a rectangular cross section of height a and width b, the moment of inertia is

$$I = \frac{a^3 b}{12} \qquad \textbf{(7-1-9)}$$

The curvature is

$$\frac{1}{\rho(x)} = \frac{\frac{\partial^2 y}{\partial x^2}}{[1+(\frac{\partial y}{\partial x})^2]^{3/2}} \approx \frac{\partial^2 y}{\partial x^2} \quad \textbf{when} \quad \frac{\partial y}{\partial x} << 1 \qquad \textbf{(7-1-10)}$$

Combine Eqns. (7-1-8) and (7-1-10), we can have

$$\frac{\partial^2 y}{\partial x^2} = \frac{M}{EI} \qquad \textbf{(7-1-11)}$$

Now consider a beam with one end clamped and the other under an applied force F and Torque M_0 as in the figure. If the beam is in equilibrium, then we must have torque balance at any point along its length, which implies that

$$M(x) = F(L-x) + M_0 \qquad \textbf{(7-1-12)}$$

We then obtain

$$\frac{\partial^2 y}{\partial x^2} = \frac{F(L-x)+M_0}{EI} \qquad \textbf{(7-1-13)}$$

from Eqn. (1). Integrating twice and applying the boundary conditions that y(0)=0 and y'(0)=0 gives

$$y(x) = \frac{1}{EI}[\frac{Fx^2}{6}(3L-x) + \frac{M_0 x^2}{2}] \qquad \textbf{(7-1-14)}$$

The angle of the beam is the first derivative of displacement with respect to position

$$\theta(x) = \frac{\partial y}{\partial x} = \frac{1}{EI}[Fx(L-\frac{x}{2}) + M_0 x] \qquad \textbf{(7-1-15)}$$

So the deflection and orientation of the tip of the beam is

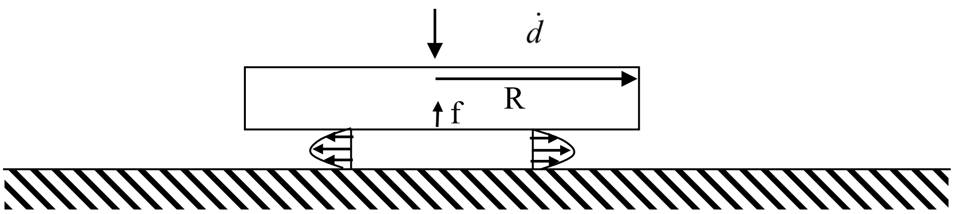
$$y(L) = \frac{1}{EI}\left[\frac{FL^3}{3} + \frac{M_0 L^2}{2}\right] \qquad \textbf{(7-1-16)}$$

$$\theta(L) = \frac{1}{EI}\left[\frac{FL^2}{2} + M_0 L\right] \qquad \textbf{(7-1-17)}$$

For the special case where end load is a pure force (no moment), the force/deflection relation is linear and can be written as F=kx. Where k is a function of the dimensions of the beam. For a rectangular beam, I=a^3b/12

$$k(a,b,L) = \frac{3EI}{L^3} = \frac{a^3 bE}{4L^3} \qquad \textbf{(7-1-18)}$$

◆ Squeeze-film damping



When the motion of a plate is perpendicular to another surfaces, and the separation between the plates is small compared to the width and length of the plates, there will be normal force on each plate due to squeeze-film damping. By making a few simplifying assumptions, a closed form solution of the Navier-Stoke equations for round plates of radius R is:

$$F = \frac{k_b}{d^3}\dot{d} \qquad k_b = \frac{3\pi\mu R^4}{2} \qquad \textbf{(7-1-19)}$$

◆ Maximum Strain

For and isotropic beam of beam thickness a, given curvature ρ, the strain ε varies linearly through the thickness of the beam, with extreme on each surface, and zero strain in the center of the beam on the neutral axis. From geometric arguments it can be shown that the magnitude of the surface strain is given by

$$\varepsilon = \frac{a}{2\rho} \qquad \textbf{(7-1-20)}$$

For a pure force applied to the end of the beam, the moment varies linearly as a function of arc length along the beam (Eqn. 7-1-12), with a maximum moment at the base of the beam M_{max}=FL which translates to maximum curvature of

$$(\frac{1}{\rho})_{\max} = \frac{FL}{EI} = \frac{3y(L)}{L^2} \qquad \textbf{(7-1-21)}$$

so the maximum strain in the beam occurs at the base and has magnitude

$$\varepsilon_{\max} = \frac{3y(L)a}{2L^2} \qquad \textbf{(7-1-22)}$$